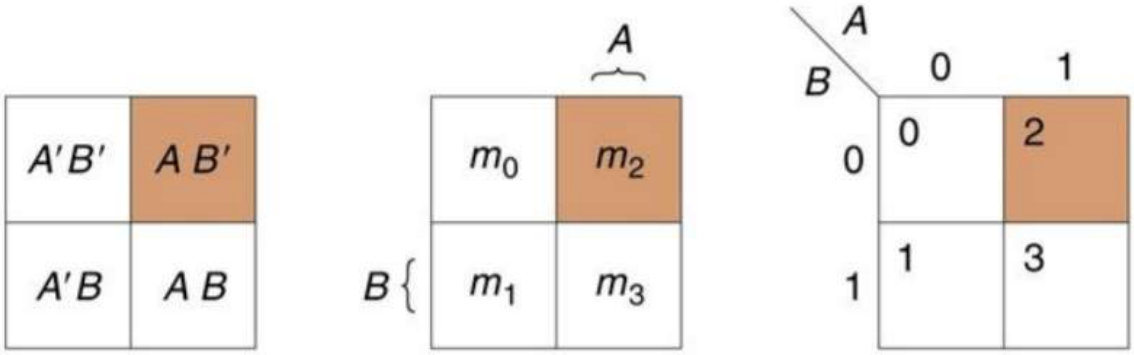
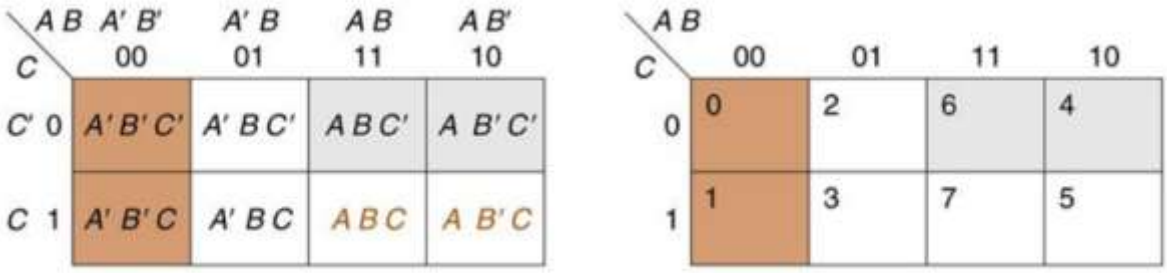
Computer System Architecture
COMP201TH
Lecture-6
Implicants, Prime Implicants, Essential Prime Implicants

- From the last lecture, we know; K map for 2, 3 and 4-variables is:



2-variable K Map



3-variable K Map

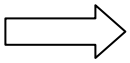
CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

CD \ AB	00	01	11	10
00	A'B'C'D'	A'BC'D'	ABC'D'	AB'C'D'
01	A'B'C'D	A'BC'D	ABC'D	AB'C'D
11	A'B'CD	A'BCD	ABCD	AB'CD
10	A'B'CD'	A'BCD'	ABCD'	AB'CD'

4-variable K map

- **Covering of a function:**
  - A switching function $f(x_1,x_2,....x_n)$ is said to cover $g(x_1,x_2,....x_n)$ denoted by "f is superset of g" if f assumes true value whenever g does.

	g	f
0	0	1
1	1	1
2	0	0
3	0	0



Whenever g is 1, f is also 1 i.e. f is having true value whenever g is true.

f g
{0,1} ⊇ {1}
Here f contains $minterm_0$ and $minterm_1$.
So, we can say:
f covers g

- **If g has x minterms and g is a function of n variables, then number of covering functions for g is**

$$2^{n}\text{-}x$$
$$2$$

- **Implicant:**
  - A group of one or more 1's which are adjacent and can be combined on a Karnaugh Map is called an implicant.
    - i.e. in this we include group of 1 1's, 2 1's, 4 1's, 8 1's, 16 1's…..2^n 1's i.e. while grouping 1's we group them in power of 2 and if single 1 is left then its also taken as a group.
  - From the point of view of the map, an implicant is a rectangle of 1,2,4,8,… 2^n 1's.
  - In other words, an implicant is a product term that can cover minterms of a function.
- **Prime Implicant:**
  - Largest possible group of 1's. In other words, the biggest group of 1's which can be circled to cover a given 1 is called a prime implicant.
  - To find prime implicants, we use all possible groups formed in K-map.
    - A minimal solution will never contain non-prime implicants.
- **Essential Prime Implicant:**
  - prime implicant in which at least there is single 1 which cannot be combined in any other way.
    - Essential prime implicants are those implicants which always appear in final solution.
  - These are those groups which cover at least one minterm that can't be covered by any other prime implicant.
- **Non Essential Prime Implicants:**
  - A prime implicant in which all of its covered 1 squares are covered by one or more other prime implicants.
  - i.e. a prime implicant where every one of its squares is part of another prime implicant.

Why do we need all these implicants?

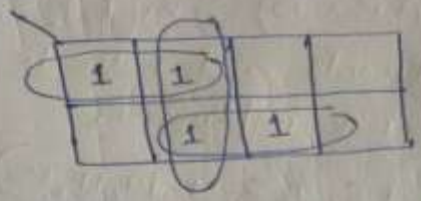
- While finding minimum SOP expressions from K map the only product term we need to care about are prime implicants.
- Essential prime implicants are the prime implicants that must be used in any min SOP expression. It may have non-essential prime implicants only if the essential prime implicants don't cover all squares.
- There can be more than one simplified SOP due to the selection of non-essential prime implicants.

Procedure for finding the SOP form a K Map:

- Step 1: Form the 2-,3-, or 4- variable K map as appropriate for the Boolean function.
- Step 2: Identify all essential prime implicants for 1s in the K map.
- Step 3: Identify non-essential prime implicants for 1s in the K map.
- Step 4: For each essential and one selected non-essential prime implicant from each set, determine the corresponding product term.
- Step 5: Form a sum-of-products with all product terms from previous steps.

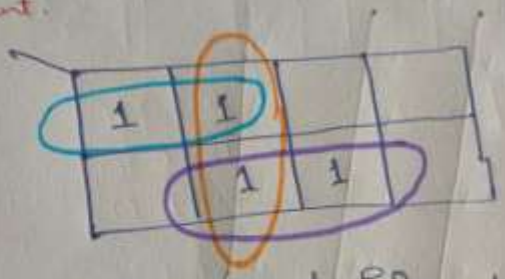
Examples:

E.g. -



⇒ No. of Prime Implicants = 3

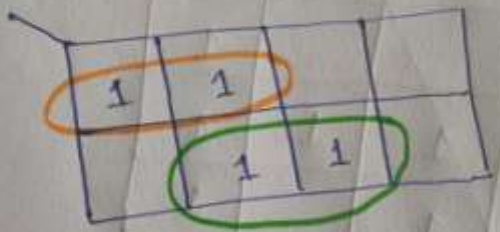
Prime implicant: The biggest group of 1s which can be circled to cover a given 1 is called a prime implicant.



⇒ = 3

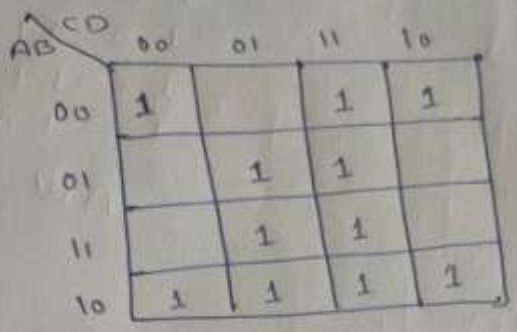
Essential Prime Implicant = 2.

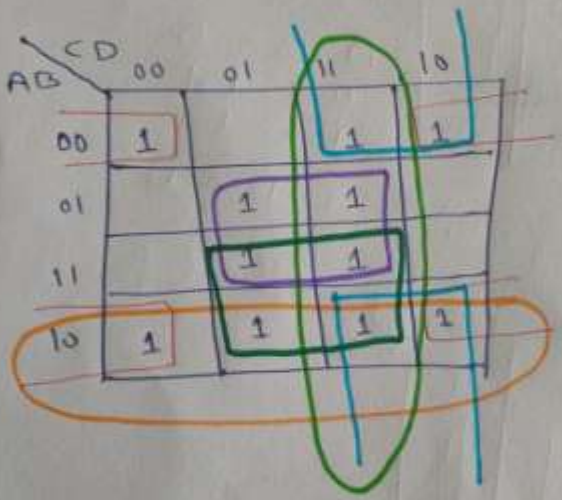
↳ those groups which cover atleast one minterm that can't be covered by any other prime implicant.



⇒ = 2

eg. Find all prime implicants:





Group 1:

A	B	C	D
0	0	0	0
0	0	1	0
1	0	0	0
1	0	0	0

B' D'

Group: 2

A	B	C	D
0	0	1	1
0	1	1	1
1	1	1	1
1	0	1	1

C D

Group: 3

A	B	C	D
0	0	1	1
0	0	1	0
1	0	1	1
1	0	1	0

B'C

Group: 4

A	B	C	D
0	1	0	1
0	1	1	1
1	1	0	1
1	1	1	1

B D

Graph: 5

A	B	C	D
1	0	0	0
1	0	0	1
1	0	1	1
1	0	1	0

AB'

Graph: 6

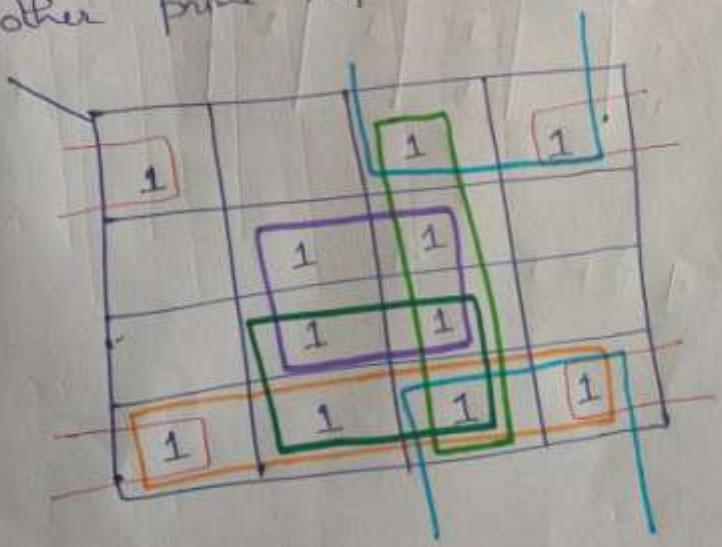
A	B	C	D
1	1	0	1
1	1	1	1
1	0	0	1
1	0	1	1

AD

∴ Number of prime implicants are: 6 & which are: $B'D'$, CD, $B'C$, BD, AB', AD

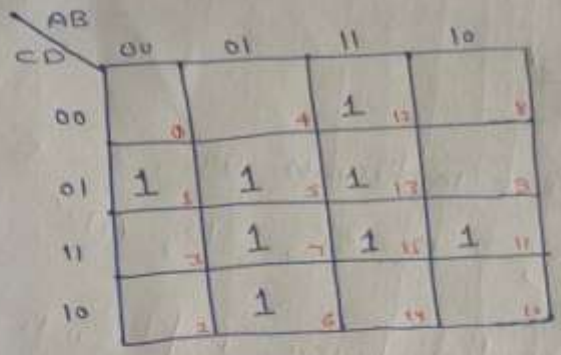
And number of essential prime implicants are: 2 & which are: $B'D'$, BD.

Essential prime implicants are those graphs which cover atleast one minterm that can't be covered by any other prime implicant.

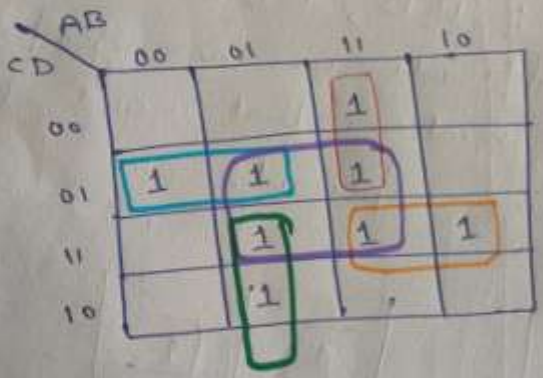


Eg. Given $F = \Sigma(1, 5, 6, 7, 11, 12, 13, 15)$; Find number of implicants, PI (prime implicants), EPI (essential prime implicants).

Sol:



No. of implicants = 8



No. of prime implicants = 5

No. of EPI = 4