Computer System Architecture
COMP201Th
Lecture: 17
Number Systems

Number systems are the technique to represent numbers in the computer system architecture. Electronic and Digital systems may use a variety of different number systems viz.:

- Binary (base 2)
- Octal (base 8)
- Decimal (base 10)
- Hexadecimal (base 16)

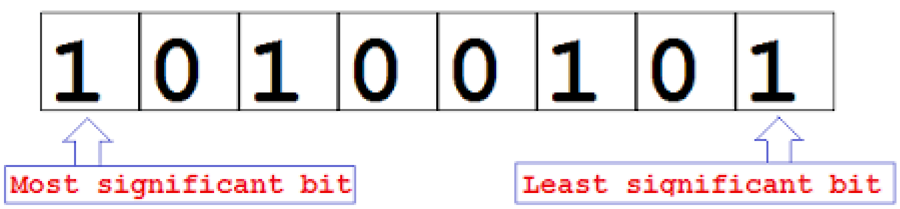
A number N in base or radix b can be written as:

$$(N)_b = d_{n-1}\ d_{n-2} \text{-----} d_1\ d_0\ .\ d_{-1}\ d_{-2} \text{----------} d_{-m}$$

In the above, d_{n-1} to d_0 is integer part, then follows a radix point and then d_{-1} to d_{-m} is fractional part.

d_{n-1} = Most Significant bit (MSB)

d_{-m} = Least Significant bit (LSB)



System	Radix	Allowable Digits
Binary	2	0,1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Conversion among Number Systems:

- **Decimal number into binary equivalent:**
  - Write down the decimal number and continually divide it by 2 to give a result and a remainder of either a “1” or “0” until the final result equals zero.

e.g. Convert $(39)_{10} = (\quad)_2$

$$(39)_{10} = (?????)_2$$

2	39		
2	19	1	LSB
2	9	1	
2	4	1	
2	2	0	
2	1	0	
	0	1	MSB

$$= \underset{\text{MSB}}{1}0011\underset{\text{LSB}}{1}$$

$$(39)_{10} = (100111)_2$$

- **Decimal Fraction to binary equivalent:**
  - Multiply the fraction by 2 keeping notice of the resulting integer and fractional part. Continue multiplying by 2 until you get a resulting fraction equal to zero. Then just write out the integer parts from the results of each multiplication.

$$(0.375)_{10} = (?????)_2$$

Fraction	Fraction *2	Remainder Fraction	Integer	
0.375	0.750	0.75	0	MSB
0.750	1.50	0.50	1	
0.50	1.00	0.00	1	LSB

$$=$$

.011

MSB LSB

- **Decimal Number to Octal Number:**

e.g. $(461)_{10} = (???)_8$

$$(461)_{10} = (???)_8$$

8	461		
8	57	5	LSD
8	7	1	
	0	7	MSD

$$= (715)_8$$

$$(461)_{10} = (715)_8$$

- **Decimal Number to Hexadecimal Number:**

e.g. $(10767)_{10} = (????)_{16}$

$$(10767)_{10} = (????)_{16}$$

16	10767	
16	672	15 =F
16	42	0
16	2	10 =A
	0	2

$$= (2A0F)_{16}$$

Binary	Hex	Decimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	A	10
1011	B	11
1100	C	12
1101	D	13
1110	E	14
1111	F	15

$$(10767)_{10} = (2A0F)_{16}$$

- **Binary to Decimal Conversion:**

e.g. $(10111)_2 = (????)_{10}$

$$(10111)_2 = (?????)_{10}$$

$$\begin{array}{ccccc} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$$=$$

$$1*2^0 + 1*2^1 + 1*2^2 + 0*2^3 + 1*2^4$$

$$=$$

$$1+2+4+16 = 23$$

- **Binary Fraction to Decimal Fraction Conversion:**

e.g. $(0.1011)_2 = (????)_{10}$

$$(0.1011)_2 = (????)_{10}$$

$$\begin{array}{ccccc} . & 1 & 0 & 1 & 1 \\ & 2^{-1} & 2^{-2} & 2^{-3} & 2^{-4} \end{array}$$

$$=$$

$$1*2^{-1} + 0*2^{-2} + 1*2^{-3} + 1*2^{-4}$$

$$=0.5+0+0.125+0.0625$$

$$=0.6875$$

$$(0.1011)_2 = (0.6875)_{10}$$

- **Binary to Octal Conversion:**
  - **Here we make group of three bits starting from LSB:**

e.g. $(1101110)_2 = (?????)_8$

$$(1101110)_2 = (?????)_8$$

$$1 \;\; 101 \;\; 110$$

$$=$$

$$001101110$$

$$=$$

$$156 = (156)_8$$

- **Binary to Hexadecimal Conversion:**
  - **Here we will make group of 4 bits starting from LSB.**

e.g.$(1101011010)_2 = (?????)_{16}$

$$(1101011010)_2 = (?????)_{16}$$

$$11 \; 0101 \; 1010$$

$$=$$

$$001101011010$$

$$=$$

$$35A = (35A)_{16}$$

- **Octal to Binary Conversion:**
    - **Here, we just write binary equivalent of the digit.**

e.g. $(527)_8 = (?????)_2$

$= (101)\ (010)\ (111)$

$= (101010111)_2$

- **Octal to Decimal Conversion:**

e.g. $(140)_8 = (????)_{10}$

$= 0*8^0 + 4*8^1 + 1*8^2$

$= (96)_{10}$

- **Octal to Hexadecimal Conversion:**
    - **First, convert the octal number to binary number.**
    - **Then, make group of four bits starting from LSB and then convert to equivalent hexadecimal.**

e.g. $(456)_8 = (????)_{16}$

$= (100)\ (101)(110)$

$=(100101110)$

Now (100101110) → make group of 4 bits starting from LSB

= (1) (0010) (1110)

=(0001) (0010) (1110)

= (1) (2) (E)

$= (12E)_{16}$

- **Hexadecimal to Binary Number:**
    - **Just write binary equivalent of each digit.**

e.g. $(4F2D)_{16} = (????)_2$

= (0100)(1111)(0010)(1101)

$=(0100111100101101)_2$

- **Hexadecimal to Octal Number:**
    - **First write the binary equivalent.**
    - **Then make group of 3 bits starting from LSB and write octal equivalent.**

e.g. $(5A)_{16}$ = (0101) (1010)

$= (01011010)_2$

= (01) (011) (010)

= (001) (011) (010)

$= 1\ 3\ 2 = (132)_8$

- **Hexadecimal to Decimal Number:**

e.g. $(1A53)_{16} = (3*16^0) + (5*16^1) + (10 * 16^2) + (1*16^3)$

= 3+80+2560+4096

= 6739