Computer System Architecture
COMP201Th
Lecture: 18
Complements

Complements are used in digital computers for **simplifying the subtraction operation** and for logical manipulation.
Two types of complements for each base r system:

- the r's complement
  - e.g. 2's complement for binary numbers
  - and 10's complement for decimal numbers
- the (r-1)'s complement
  - e.g. 1's complement for binary numbers
  - and 9's complement for decimal numbers.

(r-1)'s Complement:

Give a number N in base r having n digits, the (r-1)'s complement of N is defined as **$(r^n - 1) - N$**.

9's complement:

- e.g. for decimal numbers: r=10 and r-1 = 9 → 9's complement of N is: $(10^n-1)-N$.
  - Now, $10^n -1$ is a number represented by n 9's.
    - e.g. with n=4, 10^4= 10000 and 10^4-1=9999
  - → **the 9's complement of a decimal number is obtained by subtracting each digit from 9.**
- e.g. the 9's complement of 546700 is:
  - 999999-546700 = 453299
- Similarly, the 9's complement of 12389 is: 99999-12389 = 87610.

1's complement:

- For Binary numbers: r =2 and r-1 = 1 → so the 1's complement of N is: $(2^n-1)-N$.
  - Now, 2^n is represented by a binary number that consists of a 1 followed by n 0's.
  - → 2^n-1 is a binary number represented by n 1's
    - e.g. n =4 → $2^4 = (10000)_2$ and $2^4-1 = (1111)_2$
  - the 1's complement of a binary number is obtained by subtracting each digit from 1 → however, the subtraction of a binary digit from 1 causes the bit to change from 0 to 1 or from 1 to 0.

- → **the 1's complement of a binary number is formed by changing 1's into 0's and 0's into 1's.**

- e.g. the 1's complement of 1011001 is 0100110
- the 1's complement of 0001111 is 1110000.

The (r-1)'s complement of octal or hexadecimal numbers are obtained by subtracting each digit from 7 or F (decimal 15) respectively.

r's Complement:

The r's complement of an n-digit number N in base r is defined as **r^n - N for N≠ 0 and 0 for N = 0.**

→ the r's complement is obtained by adding 1 to the (r-1)'s complement.

e.g.

- the 10's complement of 2389 is 7610 + 1 = 7611 → which is adding 1 to the 9's complement value.
- the 2's complement of binary 101100 is 010011+1 = 010100 → i.e. adding 1 to the 1's complement value.

10's complement can also be obtained by:

- **Leaving all least significant 0's unchanged**
- **Subtracting the first non-zero least significant digit from 10.**
- **and then subtracting all higher significant digits from 9.**

E.g. The 10's complement of 246700 is 753300 and is obtained by:

- Leaving two zeros (i.e. all least significant 0's unchanged) unchanged.
- subtracting 7 from 10 (i.e. subtracting the first non-zero least significant digit from 10)
- and subtracting the other three digits from 9.

Similarly, 2's complement can also be obtained by:

- **Leaving all least significant 0's and the first 1 unchanged.**
- **Then replacing 1's by 0's and 0's by 1's in all other higher significant bits.**

e.g. The 2's complement of 1101100 is 0010100 and is obtained by:

- leaving the two low-order 0's and the first 1 unchanged.
- and then replacing 1's by 0's and 0's by 1's in the other four most significant bits.

❖ ***What if numbers have radix point?***

If the original number N contains a radix point, it should be removed temporarily to form the r's or (r-1)'s complement. The radix point is then restored to the complemented number in the same relative position.

Also remember complement of the complement restores the number to its original value.

Unsigned Integer: can represent zero and positive integers.

Signed Integer: can represent zero, positive and negative integers.

Subtraction of Unsigned Numbers:

The subtraction of two n-digit unsigned numbers M – N (N≠0) in base r can be done as follows:

- ❖ Add the minuend M to the r's complement of the subtrahend N.
- ❖ If M≥N, the sum will produce an end carry which is discarded and what is left is the result M – N.
- ❖ If M≤N, the sum does not produce an end carry and is equal to r's complement of (N-M). To obtain the answer in familiar form, take the r's complement of the sum and place a negative sign in front.

Examples:

- Subtract: 72532 – 13250 = ???

  The 10's complement of 13250 is 86750.

  Thus, M = 72532

  10's complement of N = +86750

  Sum = 159282

  Discard end carry thus answer = 59282

- Subtract: 13250 – 72532 = ?

  The 10's complement of 72532 is 27468

  Thus, M = 13250

  10's complement of 72532 N= +27468

  Sum= 40718

  There is no end carry → answer is negative 59282 = 10's complement of 40718.

Similarly for binary numbers

- Subtract X = 1010100 and Y = 1000011.

  X = 1010100

  2's complement of Y = +0111101

  Sum = 10010001

  Discard end carry so answer X-Y= 0010001

- Now, do Y-X

  Y= 1000011

2's complement of X = +0101100

Sum = 1101111

There is no end carry → answer is negative 0010001 = 2's complement of 1101111.