Computer System Architecture
COMP201TH
Lecture-3
Map Simplification (K-Map)
Part-I
(Canonical Normal Form- Minterm and Maxterm)

"I have not failed. I've just found 10,000 ways that won't work."
...Thomas A. Edison

- **Consensus Theorem/ Redundant Theorem of Boolean Algebra:**
  - **AB + A`C + BC = AB +A`C**
  - Consensus theorem states that if you have a variable in one product term and compliment of that variable in another product term and the third product term has the remaining variable from those two products then the third product term is redundant and we can eliminate this for simplification of Boolean expression.
- **Standard Form:**
  - If there exists at least one term that does not contain all variables → Boolean expression is in standard form.
  - E.g. F(A,B,C) = A` + ABC   Here B and C variables are missing in first part of Boolean expression → expression is in standard SOP form.
- **Canonical Normal Form:**
  - Each term of Boolean expression **contain all input variables** either in True form or in complement form**.**
  - In Boolean Algebra, any Boolean function can be put into:
    - Canonical **disjunctive normal form** (CDNF) or **minterm canonical form** and
    - Canonical **conjunctive normal form** (CCNF) or **maxterm canonical form.**
  - A Boolean function can be expressed algebraically from a given truth table by forming a:
    - Minterm for each combination of the variables that produces a 1 in the function and then taking the AND of all those terms.
    - Maxterm for each combination of the variables that produces a 0 in the function and then taking the OR of all those terms.

- **Minterm:**
    - A minterm of n variable **is product** of n literals in which each variable appears exactly once either in T or F form, but not in both.
        - Minterms are called products because **they are the logical AND of a set of variables.**
    - Each minterm has value 1 for exactly one combination of values of variables.
    - E.g.: ABC (111) → m7
    - A function can be written as a sum of minterms, which is referred to as a minterm expansion or a standard ***sum of products.***
- **Maxterm:**
    - A maxterm of n variables **is sum of n** literals in which each variable appears exactly once in T or F form, but not in both.
        - Maxterms are called sums because **they are the logical OR of a set of variables.**
    - Each maxterm has a value of 0 for exactly one combination of values for variables.
    - E.g.: A + B + C\` (001) → M1 (the value is 0).
        - Therefore; $\mathbf{M_i = m_i`}$
    - A function can be written as a product of maxterms, which is referred to as a maxterm expansion or a standard ***product of sums***.
- Example consider the following truth table:

A	**B**	**C**	**F**
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**1**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

- Using f =1, gives the SOP form:

  f = A\`BC + AB\`C\` + AB\`C + ABC\` + ABC
  = A\`BC + AB\` + AB
  = A\`BC + A
  = A + BC (Using Rule 1 of Simplification Law)
- Using f = 0, gives the POS form:

  f = (A + B + C) (A + B + C\`) (A + B\` + C)
  = (A +B) (A+B\`+C)
  = A + BC

Minterm Notation	**Maxterm Notation**
Minterms present in f correspond with the 1's of f in the truth table.	Maxterms present in f correspond with the 0's of f in the truth table.
f = A\`BC + AB\`C\` + AB\`C + ABC\` + ABC	f = (A + B + C) (A + B + C\`) (A + B\` + C)
The other way to represent f is :	The other way to represent f is :
$f(A,B,C) = m_3+m_4+m_5+m_6+m_7$	**$f(A,B,C) = M_0M_1M_2$**
Or	or
f(A,B,C) = **$\sum m(3,4,5,6,7)$**	f(A,B,C) = **π M(0,1,2)**
Another view:	
$f(A,B,C) = 0.m_0+0.m_1+0.m_2+1.m_3+1.m_4+1.m_5+1.m_6+1.m_7$	

**** Remember ****

Minterm (m)	$\sum$	**Sum of Product (SoP)**	**Take terms whose value is equal to 1 in the output of truth table.**	**Disjunctive Normal Form**	**A\` = 0** **A = 1**
Maxterm (M)	Π	**Product of Sums (PoS)**	**Take terms whose value is equal to 0 in the output of truth table.**	**Conjunctive Normal Form**	**A = 0** **A\` = 1**

X	Y	Z		Minterms Product Terms		Maxterms Sum Terms
0	0	0		$m_0 = \overline{X}\cdot\overline{Y}\cdot\overline{Z} = \min(\overline{X},\overline{Y},\overline{Z})$		$M_0 = X+Y+Z = \max(X,Y,Z)$
0	0	1		$m_1 = \overline{X}\cdot\overline{Y}\cdot Z = \min(\overline{X},\overline{Y},Z)$		$M_1 = X+Y+\overline{Z} = \max(X,Y,\overline{Z})$
0	1	0		$m_2 = \overline{X}\cdot Y\cdot\overline{Z} = \min(\overline{X},Y,\overline{Z})$		$M_2 = X+\overline{Y}+Z = \max(X,\overline{Y},Z)$
0	1	1		$m_3 = \overline{X}\cdot Y\cdot Z = \min(\overline{X},Y,Z)$		$M_3 = X+\overline{Y}+\overline{Z} = \max(X,\overline{Y},\overline{Z})$
1	0	0		$m_4 = X\cdot\overline{Y}\cdot\overline{Z} = \min(X,\overline{Y},\overline{Z})$		$M_4 = \overline{X}+Y+Z = \max(\overline{X},Y,Z)$
1	0	1		$m_5 = X\cdot\overline{Y}\cdot Z = \min(X,\overline{Y},Z)$		$M_5 = \overline{X}+Y+\overline{Z} = \max(\overline{X},Y,\overline{Z})$
1	1	0		$m_6 = X\cdot Y\cdot\overline{Z} = \min(X,Y,\overline{Z})$		$M_6 = \overline{X}+\overline{Y}+Z = \max(\overline{X},\overline{Y},Z)$
1	1	1		$m_7 = X\cdot Y\cdot Z = \min(X,Y,Z)$		$M_7 = \overline{X}+\overline{Y}+\overline{Z} = \max(\overline{X},\overline{Y},\overline{Z})$

Truth table representing minterm and maxterm

- Relation between **m** and **M**:
  - If the minterm expansion for f(A,B,C) = $m_3+m_4+m_5+m_6+m_7$; what is the maxterm expansion for f(A,B,C)?
  - **Choose those not present in the minterms.**
  - So, the maxterm expansion for f(A,B,C) = $M_0M_1M_2$

Q: Find the minterm expansion of following Boolean expression:

f(A,B,C,D) = A`(B`+D)+ACD`

= A`B`+A`D`+ACD`

= A`B`(C+C`)(D+D`)+A`D(B+B`)(C+C`) + ACD`(B+B`)

=A`B`C`D`+A`B`C`D+A`B`CD`+A`B`CD+A`BC`D+A`BCD+ABCD`+AB`CD`

= 0000+0001+0010+0011+0101+0111+1110+1010

= ∑m (0,1,2,3,5,7,10,14)

w	x	y	z	F
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15

Q: Express the following function into sum of minterms and product of maxterms:

$$f= (xy+z)\ (y+xz)$$

Sol:

$$f = (xy+z)\ (y+xz)$$
$$= xy.y+xy.xz+z.y+z.xz$$
$$=xy+xyz+zy+xz$$
$$=xy+yz+xz+xyz$$

Now, consider the truth table of f:

x	y	z	f	
0	0	0	0	M_0
0	0	1	0	M_1
0	1	0	0	M_2
0	1	1	1	m_3
1	0	0	0	M_4
1	0	1	1	m_5
1	1	0	1	m_6
1	1	1	1	m_7

$f = 0.0 + 0.0 +0.0 +0.0.0 = 0 = M_0$

$f = 1.0+0.1+1.1+1.0.1= 1 =m_5$

The corresponding minterms are given by:

$$f = xy+yz+xz+xyz$$
$$= m_3 + m_5 +m_6 +m_7$$
= sum of minterms i.e. sum of products

The corresponding maxterms are given by:

$$f = (M_0)(M_1)(M_2)(M_4) = (x+y+z)(x+y+z`)(x+y`+z)(x`+y+z)$$